SUPPLEMENT:

THE CALCULATOR

Its Limitations

The more a problem frightens us, the more we tend to look for a "miracle cure". Many people are frightened about mathematics; so much so, in fact, that the term "math anxiety" has become a household word. Because of this fear, people have looked to calculators as the "cure-all". A too-often-heard argument takes the form: "Why do we have to learn how to do this when it is so much easier to use the calculator?"

To help focus our answer to this argument, we shall use the following analogy. *The calculator is to the person who has to use mathematics what the typewriter is to a person who has to write a paper.* Certainly, the typewriter gives us a neat manuscript; and for the person who can type well, it also gives a quicker way for preparing the manuscript. *However, the typewriter will not help the author overcome short-comings in writing style nor fuzziness of thought.* In a similar way, the calculator can do certain computations more quickly and possibly more accurately than the traditional methods of computation, *but it, too, cannot overcome fuzzy mathematical reasoning.*

Look, for example, at the following three fill-in-the-blank questions:

(1) 40 is ____% of 80.

(2) 40% of 80 is _____.

(3) 40% of ___ is 80.

All three problems have a similar form and all use the numbers, 40, 80, and the percent symbol. *Yet if we don't understand what makes these three problems different from one another, the calculator will not help us solve these problems;* and if we do understand the three problems, the chances are that we don't even need a calculator to solve them quickly!

Let's see what the steps are:

(1) Step 1: Divide 40 by 80.

Using common fractions we have $\frac{40}{80} = \frac{1}{2}$

In decimal form it would be 0.5.

Step 2: Multiply by 100(%)

This give us $\frac{1}{2} \times 100\% = 50\%$

In decimal form: $0.5 \times 100\% = 50.0\%$ *or* 50%

So the answer to (1) is 50%.

(2) Step 1: Divide 40 by 100 to convert 40% to a fraction.

As a common fraction we get $\frac{40}{100} = \frac{2}{5}$.

As a decimal we get 0.40 or 0.4

Step 2: Multiply by 80.

As a common fraction: $\frac{2}{5} \times 80 = 2 \times (80 \div 5) = 32$

As a decimal fraction: $0.4 \times 80 = 32.0$ *or 32.*

So the answer to (2) is 32.

(3) Step 1: Divide 80 by 40.

This tells us that 1% of the number is 2

Step 2: Multiply by 100.

This tells us that 100% of the number is 200.

So the answer to (3) is 200.

So the three examples have very different answers. The arithmetic needed in each example is very elementary. *Yet unless you know what the proper steps are, the calculator will not help you in solving these three problems. And if you know what the proper steps are, the chances are that you didn't need the calculator in order to perform the necessary arithmetic.*

The point is that we can't use the calculator as a substitute for reading comprehension; and this includes our having to know certain key vocabulary words. Let's look at a few examples.

Example 1:

What is the product of 5 and 3?

The answer, of course, is 15. To do the problem we had to know that "product" meant the answer to a *multiplication* problem. If you interpreted the word "and" to mean addition and added 3 to 5 to get 8, then you got the right answer *to the wrong problem*. The calculator does not tell you that "product" indicates multiplication.

Example 2:

What must we add to 4 to get 7?

The answer is 3. Even though the command "add" was used, the wording of the problem indicates that we have to subtract. The calculator doesn't tell us this. The most common wrong way of doing this example is to add 4 and 7 to get 11.

Example 3:

Fill in the blank:

$3 \times \_\_ = 6$

This problem is the same as $\_\_\_ = 6 \div 3$, but the calculator doesn't know this; we have to tell the calculator what to do. So if you interpreted this problem as $3 \times 6 = \_\_\_$, you'd get 18 as the answer, but it's again the answer to the wrong problem.

Notice that in all of these three examples, if you knew what to do, you wouldn't need a calculator; and if you didn't know what to do, the calculator wouldn't have helped.

It is not our aim to "put down" the calculator. Rather it is our goal to emphasize that our course is designed to teach logical mathematical thought. The calculator will not replace the thought process but will facilitate the computations that develop from our logical thought. For example, suppose we wanted to find out what percent 571 is of 937. In theory, there is no difference between this problem and the problem of finding out what percent 40 is of 80. In each case we divide the first number by the second and then multiply this quotient by 100. But while the theory is the same, it is much more cumbersome to divide 571 by 937 than it is to divide 40 by 80; *at least if we're using traditional "long-hand" techniques. But with the calculator, both examples take about the same amount of time and effort.* We'll illustrate this idea after we review the various components of the typical calculator and discuss how to use the calculator.

The Parts of a Calculator:

There is no more need to have to know the theory behind a calculator in order to use one than it is to know the theory behind how an automobile is built in order to drive one. However, in driving a car you do have to know what the brakes do, what the steering wheel does, and so on. With a calculator you have to know what the "keys" or "buttons" mean in order to use them properly. Different cars have various instruments in different places. In essence, if you know how to drive one model car, you will be able to drive another model but you might first have to get used to a few technical differences. In the same way, two different calculators may operate in slightly different ways. Finally, just as car models range from economy to luxury classes, so do calculators. The simplest calculators perform the four basic operations of arithmetic while the most sophisticated behave like small programable computers.

Just as all cars have certain parts in common, so do all calculators. For example:

(1) An "On-Off" control

You have to be able to turn a calculator on or off. Some calculators have an "on-off" switch. Other calculators have "keys" (buttons) marked "on" and "off. Obviously, to use your calculator, you should be sure that it's in the "on" position.

(2) A "Display" Portion

You have to be able to see what numbers you've put into the calculator. This display area is usually a glassed-in narrow rectangle at the top of the calculator. Reading the display allows you to check whether you've entered the correct numbers as well as to read the correct answer.

(3) A Power Source

You can't get something for nothing. There must be a source of energy, usually a battery, to power your calculator. To check whether the battery is working, switch the calculator on. A "0" should then appear in the display area.

(4) The Eleven Place Value Keys.

The calculator uses the decimal system. Hence there will be keys for each of the ten digits 0,1,2,3,4,5,6,7,8, and 9; as well as a key that represents the decimal point. To enter a decimal fraction (including whole numbers) press the digit keys in the proper order.

For example, suppose we want to put the number 23.4 into the calculator.

<u>Step 1</u>: Press the "2" key.

The display will now show: 2

<u>Step 2</u>: Press the "3" key.

The display will now show: 23

<u>Step 3</u>: Press the "." key.

The display will either read the same or else: 23.
The "dot" stands for a decimal point, not a period.

<u>Step 4:</u> Press the "4" key.

The display will now show: 23.4

We use this procedure for any decimal fraction *provided that the number of digits doesn't exceed the capacity of the display.* My calculator can only display 8 digits.

(5) The Basic Arithmetic Keys

Every calculator has at least the following five keys:

(i) The addition key, denoted by "+"

(ii) The subtraction key, denoted by "-"

(iii) The multiplication key, denoted by "X"

(iv) The division key, denoted by "÷"

(v) The equality key, denoted by "="

For example, suppose we want to add 23.4 and 7.89.

Step 1: Enter 23.4

That is, press in succession, "2", "3", ".", and "4".

Step 2: Press the addition key.

That is, press "+". There will be no change in the display.

Step 3: Enter 7.89

That is, press in succession, "7", ".", "8", and "9". You can check whether you were accurate by looking at the display which should now read: 7.89

Step 4: Press the equality key.

That is, press "=".
The display should now show: 31.29
which is the desired sum.

In general, the equality key is the one that tells the calculator to display the answer to whatever arithmetical operation you've just performed.

For the most part to perform any of the four basic operations on a pair of numbers, we enter everything in the given order. Let's look at a few examples.

Using the calculator, solye the following problems:

Problem 1:

345.78 - 9.234 = ______

		display
Step 1:	Enter 345.78	*345.78*
Step 2:	Press the "-" key.	*345.78*
Step 3:	Enter 9.234	*9.234*
Step 4:	Press the "=" key.	*336.546*

Note 1: *When you use the calculator you do not have to allign the decimal points. The calculator does this automatically.*

Note 2: *Make sure you understand how to translate word problems into the proper calculator language. For example, if the problem asks us to subtract 9.234 from 345.78, we must enter 345.78 first even though it is listed after 9.234 in the problem.*

Problem 2:

23.4 ÷ 9.99 = ______

		display
Step 1:	Enter 23.4	*23.4*
Step 2:	Press the "÷" key.	*23.4*
Step 3:	Enter 9.99	*9.99*
Step 4:	Press the "=" key.	*2.3423423*

Note 1: *If the display were greater we would notice that the quotient repeated the cycle "342" endlessly. But the display only shows 8 digits. In other words, if our answer is a non-terminating repeating decimal, the display will round off the answer to 8 digits.*

Note 2: *Again, beware of the wording of a problem. Had we been asked:*

$$9.99 \times \_\_\_\_ = 23.4$$

we'd have to be able to translate this into the form:

$$\_\_\_\_ = 23.4 \div 9.99$$

if we wanted to get the correct answer on the calculator.

Note 3: *Division doesn't have the commutative property. You must be careful, for example, not to confuse 23.4 ÷ 9.99 with 9.99 ÷ 23.4 One quotient is the reciprocal of the other.*

Note 4: *The calculator converts all rational numbers into decimal fractions. If we want the answer in terms of common fractions or mixed numbers, we have to translate the calculator's answer. This type of problem is addressed in the next example.*

Problem 3:

What is the remainder when 234,567 is divided by 683?

Step 1: Translate the problem into the proper arithmetic statement. Remember in this problem that 234,567 is the dividend and 683 is the divisor. So the problem should read:

$$234{,}567 \div 683$$

Step 2: Enter 234,567

Step 3: Press the "÷" key.

Step 4: Enter 683

Step 5: Press the "=" key.

The display will read: 343.43631

But be careful how you interpret this quotient. What the answer tells us is that 234,567 is more than the 343rd multiple of 683 but less than the 344th multiple of 683. Using the calculator we can compute what the 343rd multiple of 683 is. Namely it is 683 X 343. So we enter 683, press the multiplication key, enter 343, and press the equality key. We get 234,269 (we have to supply our own commas). We then subtract 234,269 from 234,567 to get:

234,567 - 234,269 = 298.

In more conventional language, we have:

$$\left.\begin{array}{r} 343 \\ 683\overline{)234{,}567} \\ \underline{-234{,}269} \\ 298 \end{array}\right\}$$

Using the calculator, we get 343 as the quotient in one step. We then multiply 343 by 683 to get 234,269; and we subtract this from 234,567 to get 298 as the remainder.

Depending on the price we're willing to pay for the calculator, there are many other operations that we can do conveniently. However, for the purposes of this course, all we need is one additional key on our calculator. As we emphasize in Module 11 there are many times when we want to square a number. Recall that to square a number means that we want to multiply a number by itself.

Problem 4:

What is the square of 2.315?

To do this problem requires that we multiply 2.315 by itself.

That is:

Step 1: Enter 2.315

Step 2: Press the "X" key.

Step 3: Enter 2.315.

Step 4: Press the "=" key.

The answer is 5.359225 and this is considerably less tedious than multiplying "long-hand".

Note:

Some calculators have a squaring key, denoted by "x^2" If your calculator has this key there is an easier way to find the square of 2.315. Namely enter 2.315 and simply press the "x^2" key. 5.359225 immediately appears on our display.

While the squaring key is somewhat of a luxury, the square-root key, denoted by "$\sqrt{x}$" is very important. To see why, let's look at Problem 4 from a different point of view:

Problem 5:

What number must we square to get 5.359225 as the answer?

This problem is asking us to find the number that goes into both blanks in:

$$\_\_\_\_ \times \_\_\_\_ = 5.359225$$

The square-root key gives us the answer at once. Namely:

Step 1: Enter 5.359225

Step 2: Press the square-root key

The answer, which now appears immediately in the display, is 2.315.

In summary, for the needs of our course, we need a calculator with the capacity to add, subtract, multiply, divide, and take square roots (Taking square roots is particularly tedious if we use traditional long-hand method). If you intend to go into mathematics beyond arithmetic it might be advisable to get a more sophisticated calculator. The difference in price between the basic calculators and the more sophisticated ones is so small that it is probably best to start with a more advanced calculator.

Let's discuss a few of the other possible keys you might want on your calculator.

(6) The Power Key, denoted by "y^x".

Sometimes, such as when we calculate compound interest, we want to raise a number to a rather high whole number power. For example, We might want to know the value of 1.06^8. That is, suppose we want to know the product of eight 1.06's. We could repeatedly use the calculator to compute the product:

1.06 X 1.06 X 1.06 X 1.06 X 1.06 X 1.06 X 1.06 X 1.06

But with a "y^x" key we get the answer much more rapidly.

Namely:

Step 1: Enter 1.06

Step 2: Press the "y^x" key

Step 3: Enter 8

Step 4: Press the "=" key.

Appearing on the display of my calculator is the answer, 1.5938481. *This answer can't possibly be exact because multiplying 1.06 by itself any number of times gives us a number whose last digit has to be 6. The answer we get on the calculator is rounded off to 8 digits. But for most applications this answer will be sufficiently accurate.*

(7) Grouping Symbol Keys, Denoted by "(" and ")"

Let's suppose we wanted to compute the value of

9.63 - (4.81 - 2.9786)

on our calculator.

Doing what is inside the parentheses first:

Step 1: Enter 4.81

Step 2: Press the "-" key

Step 3: Enter 2.9786

Step 4: Press the "=" key

Step 5: Record the answer: 1.8314

Step 6: Enter 9.63

Step 7: Press the "-" key

Step 8: Enter 1.8314 from Step 5

Step 9: Press the "=" key to get 7.7986 as the answer.

But now suppose our calculator has the grouping symbol keys. We can then do the problem without having to record any preliminary step. Namely:

		What we're tabulating
Step 1:	Enter 9.63	*9.63*
Step 2:	Press the "-" key	*9.63 -*
Step 3:	Press the "(" key	*9.63 - (*
Step 4:	Enter 4.81	*9.63 - (4.81*
Step 5:	Press the "-" key	*9.63 - (4.81 -*
Step 6:	Enter 2.9786	*9.63 - (4.81 - 2.9786*
Step 7:	Press the ")" key	*9.63 - (4.81 - 2.9786)*

Step 8: Press the "=" key to get 7.7986 as the answer

(8) The Reciprocal Key, Denoted by "$\frac{1}{x}$"

We have already learned that to divide by a number, we can multiply by its reciprocal. Suppose we want the reciprocal of 0.4

Using the calculator we have:

Step 1: Enter 1

Step 2: Press the "÷" key

Step 3: Enter 0.4

Step 4: Press the "=" key to get 2.5 as the answer

That is, the reciprocal of 0.4 is 1 ÷ 0.4. More generally, the reciprocal of any non-zero number x is given by 1 ÷ x, and this is often written as $\frac{1}{x}$.

Of course, if the number is given as a common fraction, we simply invert the fraction to get the reciprocal. For example, the reciprocal of $\frac{2}{5}$(0.4) is $\frac{5}{2}$(2.5). But if the number is given in decimal form, we must divide 1 by it to get the reciprocal.

If your calculator has the reciprocal key, there is a very quick way to get the reciprocal. For example the two steps for finding the reciprocal of 0.4 are:

Step 1: Enter 0.4

Step 2: Press the "$\frac{1}{x}$" key and 2.5 appears as the answer.

Some calculators come with a percent key, but this, too, is a luxury for which we have little need. Namely to find, say, 27% of a number; we can enter 27 and divide by 100 to get the equivalent of 27%. At any rate, we have no need to discuss other keys at this point in our study of arithmetic. *Beginning with the solutions for Self-Test 9, we'll show how to solve each problem had we wished to use the calculator to do the computations.*